

FAST TRACK COMMUNICATION

Current–flux characteristics in mesoscopic non-superconducting rings

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Online at stacks.iop.org/JPhysCM/22/422201**Abstract**

We propose four different mechanisms responsible for the paramagnetic or diamagnetic persistent currents in normal metal rings and determine the circumstances for changes of the current from paramagnetic to diamagnetic and vice versa. This might qualitatively reproduce the experimental results of Bluhm *et al* (2009 *Phys. Rev. Lett.* **102** 136802).

1. Introduction

In the absence of an applied voltage, an electrical current induced in normal (i.e. not superconducting) metal rings rapidly decays due to dissipation processes. However, if the radius of the ring is small enough (below microns) and the temperature of the system is below 1 K, quantum effects start to play a distinct role. Under the right circumstances electrons in the ring are able to preserve its coherence which in turn results in a persistent (dissipationless) current induced by the static applied magnetic field.

The existence of persistent currents in metallic rings was predicted by Hund in 1938 [1]. More than 30 years later Bloch [2] and Kulik [3] confirmed this prediction by means of quantum-mechanical models. Strong interest in the physics of mesoscopic rings arose after the 1983 paper [4] where the authors showed that persistent currents could flow even in the presence of disorder. Experiments on persistent currents have produced a number of confusing results in apparent contradiction with theory and even amongst the experiments themselves (e.g. the response of the rings was 10–200 larger than theoretically predicted) [5]. We have observed continued controversy on persistent currents for nearly twenty years. Recently, two groups [6, 7] have developed new different techniques which allow measurements to be made a full order of magnitude more precise than any previous attempts. Both experiments confirm to a high degree the physics theory regarding the behavior of persistent currents. The group of Moler [6] employed a scanning technique (a SQUID microscope) and measured the magnetic response

of 33 individual mesoscopic gold rings. Each ring was scanned individually, unlike past experiments on persistent currents conducted by other groups. In total the rings were scanned approximately ten million times. The team of Harris [7] developed an alternative measurement scheme. The aluminum rings were deposited on a cantilever used as a torque magnetometer whose vibration frequency can be precisely monitored. From the frequency shift caused by the magnetic flux, the researchers could deduce the current with a precision of two orders of magnitude greater than was possible in the past. They studied several different cantilevers decorated with a single aluminum ring or arrays of hundreds or thousands of identical aluminum rings. The rings on different cantilevers had radiuses of 308–793 nm.

Persistent currents are highly sensitive to a variety of factors. It is clearly visible in the experimental data shown in panel (b) of figure 2 in [6] that for nominally identical samples the observed response is paramagnetic (e.g. for ring 1) or diamagnetic (e.g. for ring 2). The goal of this work is to constitute the possible mechanisms responsible for diamagnetic or paramagnetic responses of the metallic mesoscopic rings and determine the conditions for which the transition from paramagnetic to diamagnetic current (and vice versa) is able to occur.

This paper is arranged as follows. In section 2, we start with the presentation of the model for the flux dynamics in the rings. Next, in section 3, we identify the operating conditions on which similar rings can exhibit opposite responses. We finalize the paper with conclusions in section 4.

2. The model for flux dynamics of mesoscopic rings

Let us consider one normal metal ring. At a low temperature, the ring threaded by a magnetic flux can display a persistent current I_P carried by phase-coherent electrons. The circumference of the ring has to be smaller than the electrons' phase coherence length. This typically limits the sample size to below micrometers and the temperature to below 1 K. However, at temperature $T > 0$, a part of the electrons in the ring loses its phase coherence due to thermal fluctuations and this part of the electrons constitutes a dissipative Ohmic current I_R associated with the resistance R . The actual magnetic flux ϕ induced by the current flowing in the ring is given by the relation

$$\phi = \phi_e + LI = \phi_e + L(I_P + I_R), \quad (1)$$

where ϕ_e is the magnetic flux generated by the external constant magnetic field, I denotes the total current flowing in the ring which is a sum of the persistent current I_P and the Ohmic current I_R , and L stands for the self-inductance of the ring. The persistent current I_P is a function of the magnetic flux ϕ and depends on the parity of the number of coherent electrons. Let p denote the probability of an even number of coherent electrons and $1 - p$ be the probability of an odd number of coherent electrons. Then the persistent current can be expressed in the form [9]

$$I_P = I_P(\phi) = p I_E(\phi) + (1 - p) I_O(\phi), \quad (2)$$

where

$$I_E(\phi) = I_O(\phi + \phi_0/2) = I_0 \sum_{n=1}^{\infty} A_n(T/T^*) \times \cos(nk_F l) \sin(2n\pi\phi/\phi_0), \quad (3)$$

where I_0 is the maximal current at zero temperature. The temperature dependent amplitudes are determined by the relation [9]

$$A_n(T/T^*) = \frac{4T}{\pi T^*} \frac{\exp(-nT/T^*)}{1 - \exp(-2nT/T^*)}, \quad (4)$$

where the characteristic temperature T^* is proportional to the energy gap Δ_F at the Fermi surface, k_F is the Fermi momentum and l is the circumference of the ring. If the number N of electrons is fixed then $k_F = \pi N/l$ and the persistent current takes the form

$$I_P(\phi) = I_0 \sum_{n=1}^{\infty} A_n(T/T^*) \sin(2n\pi\phi/\phi_0) \times [p + (-1)^n(1 - p)]. \quad (5)$$

The Ohmic (dissipative) current $I_R = I_R(\phi)$, according to Ohm's law and Lenz's rule, assumes the form

$$I_R = -\frac{1}{R} \frac{d\phi}{dt} + \sqrt{2D} \Gamma(t). \quad (6)$$

Herein, $\Gamma(t)$ describes thermal fluctuations of intensity D . As a result, from equations (1) to (6) we can obtain the evolution equation for the magnetic flux ϕ . Its dimensionless form reads (details of the derivation are presented in [8])

$$\frac{dx}{ds} = -\frac{dV(x)}{dx} + \sqrt{2D_\lambda(x)} \Gamma(t), \quad (7)$$

where $x = \phi/\phi_0$ is the rescaled magnetic flux and $\phi_0 = h/2e$ is the flux quantum. The rescaled time $s = t/\tau_0$, where the characteristic time $\tau_0 = L/R$ is the inductive time of the ring. For a typical mesoscopic ring, L/R is in the picosecond range. The function

$$V(x) = \frac{1}{2}(x - x_e)^2 + B(x), \quad (8)$$

where $x_e = \phi_e/\phi_0$ and

$$B(x) = \alpha \sum_{n=1}^{\infty} \frac{A_n(T_0)}{2n\pi} \cos(2n\pi x) [p + (-1)^n(1 - p)], \quad (9)$$

where $\alpha = LI_0/\phi_0$ and $T_0 = T/T^*$.

Thermal equilibrium fluctuations are modeled by δ -correlated Gaussian white noise $\Gamma(t)$ of the zero mean. Classically this situation holds true in many cases. When the temperature is lowered, however, the quantum nature of the thermal fluctuations becomes important and starts to play a role. Therefore, the standard diffusion coefficient $D_0 = k_B T/R$ (k_B denotes the Boltzmann constant) is modified due to quantum effects like tunneling, quantum reflections and purely quantum fluctuations [10–12]. The modified diffusion coefficient D_λ assumes the form [11, 8]

$$D_\lambda(x) = \frac{\beta^{-1}}{1 - \lambda\beta V''(x)} \quad (10)$$

with $\beta^{-1} = k_B T/2E_m = k_0 T_0$, the elementary magnetic flux energy $E_m = \phi_0^2/2L$ and $k_0 = k_B T^*/2E_m$ is the ratio of two characteristic energies. The prime denotes differentiation with respect to x . The dimensionless quantum correction parameter [10]

$$\lambda = \lambda_0 \left[\gamma + \Psi \left(1 + \frac{\epsilon}{T_0} \right) \right], \quad \lambda_0 = \frac{\hbar R}{\pi \phi_0^2}, \quad \epsilon = \frac{\hbar/2\pi C R}{k_B T^*}, \quad (11)$$

where the psi function $\Psi(z)$ is the logarithmic derivative of the gamma function, $\gamma \simeq 0.5772$ is the Euler gamma constant and C is capacitance of the system related to charging effects. The parameter λ characterizes quantum corrections to classical thermal fluctuations and can be formulated as the difference between the quantum and classical fluctuations of the dimensionless flux,

$$\lambda = \langle x^2 \rangle_q - \langle x^2 \rangle_c \quad (12)$$

where $\langle \cdot \rangle$ denotes the thermal equilibrium average, and the subscripts q and c refer to the quantum and classical cases, respectively. Let us remember that the diffusion coefficient $D_\lambda(x)$ cannot be negative and therefore the parameter λ has to be chosen small enough to ensure its non-negativeness for any argument. Because in equation (7) the noise term contains the multiplicative white noise $\Gamma(t)$, it is important to stress that equation (7) has to be interpreted in the Ito sense [13]. Therefore, the corresponding Fokker–Planck equation for the time evolution of the probability density $P(x, t)$ reads [13]

$$\frac{\partial}{\partial t} P(x, t) = \frac{\partial}{\partial x} \left[\frac{dV(x)}{dx} P(x, t) \right] + \frac{\partial^2}{\partial x^2} [D_\lambda(x) P(x, t)]. \quad (13)$$

The average stationary dimensionless current i flowing in the ring can be calculated from equation (1):

$$i = \langle x \rangle - x_e, \quad i = \langle I \rangle L / \phi_0 \quad (14)$$

and the average stationary magnetic flux $\langle x \rangle$ is calculated from the relation

$$\langle x \rangle = \int_{-\infty}^{\infty} x P(x) dx, \quad (15)$$

where $P(x) = \lim_{t \rightarrow \infty} P(x, t)$ is a stationary probability density being a solution of the Fokker–Planck equation (13) for $\partial P(x, t) / \partial t = 0$ and zero stationary probability current. It has the form

$$P(x) = N_0 D_\lambda^{-1}(x) \exp[-\Psi_\lambda(x)], \quad (16)$$

where N_0 is the normalization constant and the generalized thermodynamic potential $\Psi_\lambda(x)$ reads

$$\Psi_\lambda(x) = \int \frac{dV(x)}{dx} D_\lambda^{-1}(x) dx = \beta V(x) - \frac{1}{2} \lambda \beta^2 [V'(x)]^2. \quad (17)$$

In the case when the thermal fluctuations can be treated classically, i.e. when $\lambda = 0$, the stationary state $P(x)$ is described by the Boltzmann distribution. When the thermal fluctuations have to be considered as quantum fluctuations, the stationary state is still a thermal equilibrium state but is now described by the non-Boltzmann distribution (16) with the x -dependence of the diffusion coefficient $D_\lambda(x)$.

3. Current–flux characteristics

We are interested in the current–flux characteristics, i.e. in dependence of the stationary current $i = i(x_e)$ on the applied magnetic flux x_e . To this aim we exploit equations (14)–(17). Note that the external flux x_e enters equation (14) and the remaining equations via the potential (8) which occurs both in the diffusion function $D_\lambda(x)$ and the generalized thermodynamical potential $\Psi_\lambda(x)$. Because the persistent current is a periodic function of the magnetic flux, we consider the current–flux characteristics on the unit interval of x_e . We present four visually similar sets of the current–flux characteristics with various setups to demonstrate the sensitivity of the persistent currents to some subtle effects. This could explain the different response of the nominally identical metal rings to the magnetic flux presented in the recent experimental work [6]. The measured persistent current of 15 nominally identical rings with a radius $R = 0.67 \mu\text{m}$ exhibits both the paramagnetic and diamagnetic responses. In panel (b) of figure 2 in [6], one can easily notice that in the linear response regime, i.e. for values of the external flux close to zero, the susceptibility defined as the ratio of the average persistent current to the external flux sometimes has a positive slope indicating the paramagnetic current (see the current for the rings 1, 3, 4, 10, 11, 12, 15 in figure 2(b) in [6]) and sometimes has a negative slope exhibiting the diamagnetic response (see the current for the rings 2, 5, 6, 7, 8, 9, 13, 14 in figure 2(b) in [6]).

Figure 1 illustrates possible current–flux characteristics. All parameters are chosen arbitrarily as we do not aim to compare our results with the experimental data. The goal of this work is rather to constitute possible mechanisms responsible for diamagnetic or paramagnetic responses. Nevertheless, all curves corresponding to the paramagnetic currents (blue solid lines) in all panels look very similar to the experimental curve shown in figure S6(A) in the supporting online material [14] of the paper [7].

In panel (a) we depict the current–flux characteristics in the regime of classical thermal fluctuations (i.e. when $\lambda = 0$) for three different temperatures. For the lowest temperature $T_0 = 0.6$ (blue solid line) the situation reveals the paramagnetic response of the ring to the applied magnetic flux. For the higher temperature $T_0 = 0.7$ (red dashed–dotted line) the response is diamagnetic. The cross-temperature below which the response is paramagnetic and above which the response is diamagnetic was recognized numerically at the value of $T_0^C = 0.6423$ (see the green dashed line and the inset of panel (a) for details).

Panel (b) presents the ring reaction to the external stimulus for three values of the structure parameter $k_0 = k_B T^* / 2E_m$. It stands for the ratio of the two characteristic energies, the thermal energy $k_B T^* / 2$ to the magnetic energy $E_m = \phi_0^2 / 2L$. It is instructive to define k_0 in the alternative way as $k_0 = LI_0 / \phi_0$. At the value of $k_0 = 0.09$ the response is paramagnetic (blue solid line), for $k_0 = 0.12$ it is diamagnetic (red dashed–dotted line) and again we have numerically identified the cross-value at the level of $k_0^C = 0.1052$ (green dashed line). We note that the ratio of two parameters $r_\alpha = \alpha / k_0 = i_0 / I_0 \sim l_e / l$ describes the physical properties of the metal ring [15]. Here l_e is the elastic mean free path of the electron in the ring and l stands for the circumference of the ring. For the multichannel rings the above ratio is greater than 1 for the ballistic regime and smaller than 1 in the diffusive one. In the presented collection of the parameters the cross-value of the para-to-diamagnetic response lies slightly below the value of unity, i.e. at $r_\alpha^C \simeq 0.95$. It means that this border value lies in the diffusive regime and all rings designated by the higher values of k_0 showing the diamagnetic susceptibility will also lie in the same diffusive regime.

It is well known that the most straightforward way of turning the susceptibility from diamagnetic to paramagnetic is to change the probability p of an even number of coherent electrons in the ring. In panel (c) we present the current for three different rings with probabilities $p = 0.5$ (blue solid line), 0.455 (green dashed line) and 0.5 (red dashed–dotted line) exhibiting the paramagnetic, zero and diamagnetic susceptibility, respectively.

In the last panel (d) we focus on the influence of the quantumness of the thermal fluctuations on the persistent currents. The comparison of the classical thermal fluctuations with $\lambda_0 = 0$ and corresponding paramagnetic current (blue solid line) with two values of the non-zero quantum noise parameter $\lambda_0^C = 0.00161$ exhibiting zero magnetic susceptibility (green dashed line) and $\lambda_0 = 0.002$ with the diamagnetic response (red dashed–dotted line) is presented. As shown, the sign of the current in the vicinity of zero

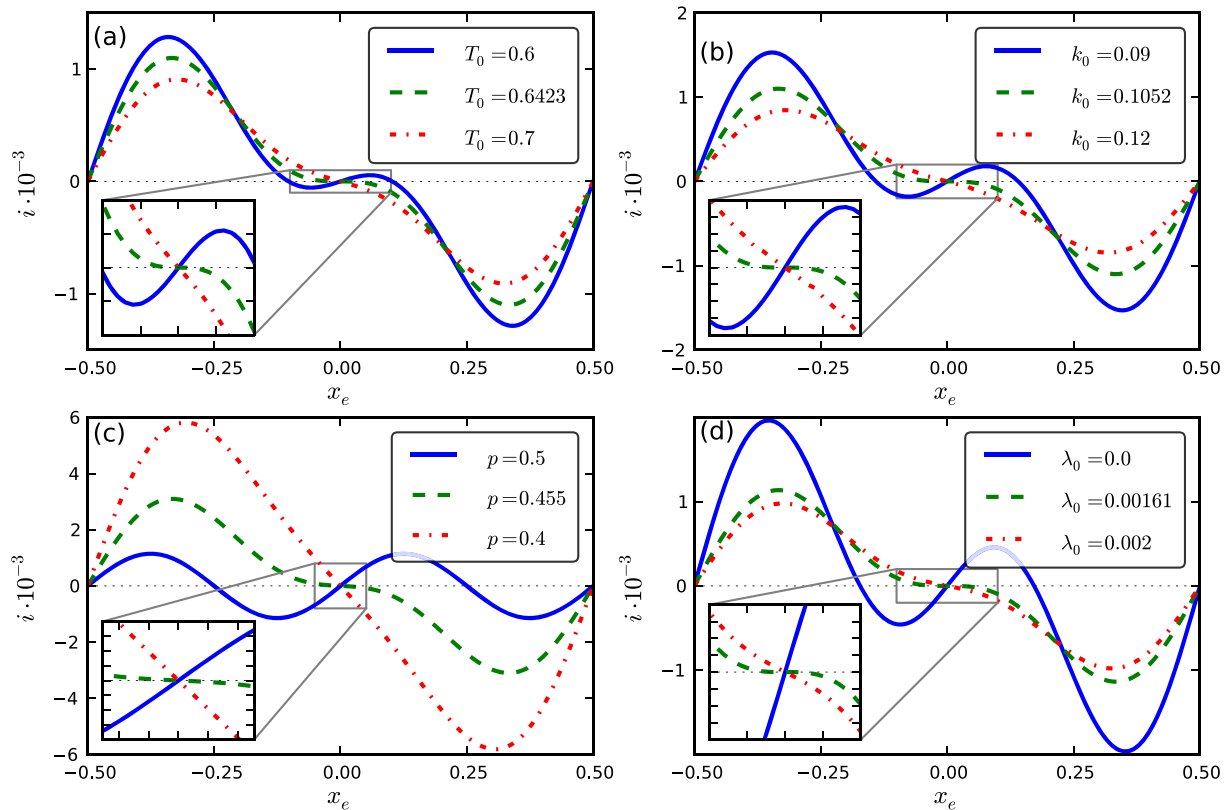


Figure 1. The stationary averaged velocity versus the external magnetic flux x_e . In panel (a) we show current–flux characteristics in the classical regime for three different temperatures $T_0 = 0.6, 0.6423, 0.7$. Panel (b) also presents the classical regime for three different structure constants $k_0 = 0.09, 0.1052, 0.12$. Panel (c) exhibits again the persistent current for a ring working in the classical regime for three values of the probability $p = 0.4, 0.455, 0.5$ of an even number of coherent electrons in the ring. Finally panel (d) reveals the influence of the quantum parameter $\lambda_0 = 0$ (indicating the classical regime), $0.00161, 0.002$ (quantum regime). Blue (solid) lines mark the paramagnetic response to the external stimulus, green (dashed) lines denote the situation where the magnetic susceptibility is zero and finally red (dashed–dotted) lines indicate the diamagnetic susceptibility of the normal metal meso-ring around zero external magnetic flux (see insets for details). The parameters not given explicitly are set as follows: $T = 0.5, p = 0.48, k = 0.08, \alpha = 0.1$.

(This figure is in colour only in the electronic version)

magnetic field can easily be affected by small perturbations of the quantum correction parameter λ .

We analyzed the averaged current in dependence on the magnetic flux and system parameters. Another important aspect is related to the current fluctuations. For a one-dimensional ring (with a quantum dot) coupled capacitively to an external dissipative impedance, zero-point quantum fluctuations lead to a strong suppression of the persistent current with decreasing external impedance and the current fluctuations can exceed the average persistent current [16]. The dependence of the current and its fluctuations on the coupling of the ring to environment is discussed in [17–19].

4. Conclusions

We have proposed four mechanisms that can lead to either diamagnetic or paramagnetic currents in the metal rings. Two of them are related to the physical properties of the metal rings. These are the structure parameter k_0 together with the probability p of an even number of coherent electrons in the ring. Another two can be adjusted by tuning the temperature of the system. In the experiment [7], the temperature uncertainty

is 7% [14]. In all the presented cases, the small change of the control parameter causes the reversal of the susceptibility—from paramagnetic to diamagnetic or vice versa. Currently in experiments on mesoscopic rings it is impossible to justify which system parameters are responsible for the observed responses. The fact that scientists are able to perform the measurements on single separated rings is a great success and a milestone in the present state-of-the-art. Now, experimental physicists are able to prepare experiments in the desired and more precise conditions and test our findings.

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