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Transmission of magnetic signals in noisy mesorings

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Abstract. A linear array of co-planar magnetically coupled mesoscopic non-superconducting rings is considered. Propagation of an alternating magnetic flux driving one of the rings can be effectively controlled by means of the properties of rings. We report the observation of the phase lag, vibrational amplification and rectification of periodic signals.

Keywords: mesoscopic systems (theory), driven diffusive systems (theory)

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Contents

1. Introduction

Transmission of signals on a macroscale is a well known and solved problem of electrical engineering and radio-technology [1]. The intensive growth of the branch of physics commonly known as nanotechnology has resulted in the highly non-trivial problem of implementing systems which are a stage of both classical and quantum phenomena. Quantum phenomena manifested at a mesoscopic level have attracted considerable attention from both theorists and experimentalists. As a consistent description of the quantum–classical hybrids has not been established so far, there are still many problems in this field which remain unsolved. One of the 'mysterious' mesosystems is that of nanorings made of non-superconducting materials. Such rings, due to the quantum size effect, can accommodate currents of phase coherent electrons and as a result exhibit equilibrium persistent currents. The quantitative description of these currents is a well known failure of typical theoretical descriptions. Experimental data show that persistent currents are about two orders of magnitude larger then predicted by theoretical models [2]. Nevertheless, systems of cylindrical symmetry offer a rich spectrum of practical applications. More recent studies show that they are potential candidates for flux qubit implementations based on non-superconducting materials [3] and they can serve as an architecture for the processing of quantum information.

In this paper, we apply the semi-phenomenological modeling proposed recently in [4, 5] to describe some particular features of a linearly ordered array of such rings as a natural candidate system for transmitting magnetic fluxes. We limit our attention to a particular class of the array: the sets of three rings. The first of them is subject to a periodically modulated magnetic flux. This signal is effectively transmitted to the rest of the array. The information loss depends not only on the amount of noise related to the temperature of the system but also on the particular features of mesorings. We focus our attention on three particularly spectacular properties of the signal transmission via mesorings. The first is the possibility of phase attenuation of the signal, i.e. the non-vanishing phase shift between the input and the output signal. The second is the noise-assisted amplification

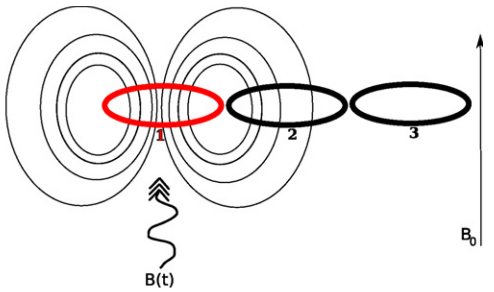

Figure 1. Schematic cartoon of three co-planar rings driven by a static magnetic field B_0. Additionally a time-dependent magnetic field $B(t)$ is acting on the first ring. The lines of magnetic flux produced by the currents flowing in the second and third rings are omitted for clarity.

of the periodic signal transmitted by the bistable system. The last but clearly not the least problem considered below is the possibility of using mesorings in design of a rectifier of periodic magnetic signals. None of the above mentioned problems is restricted to basic research, but rather may be expected to be implemented by means of nanotechnology.

2. Co-planar chains of rings

Let us assume that the system of three mesoscopic rings is placed in a co-planar one-dimensional array (see figure 1). Such rings can accommodate currents. At vanishing temperature in ideally clean materials those currents are predicted to be persistent. In real systems, some electrons lose their coherence due to thermal fluctuations. As a result, the dissipative Ohmic current coexists with the coherent current. Being close enough, such rings interact with each other by means of mutual inductance and self-inductively produce magnetic fluxes. The first ring is additionally driven by some external magnetic field. We consider the regime of a linear response. Therefore, the magnetic flux ϕ_i in the ith ring and the current I_k in the kth ring are linearly related, namely,

$$\begin{aligned} &\phi_i + \phi_{\text{ext}}(t) = \sum_{k=1}^{3} \mathcal{M}_{ik} I_k, \qquad i = 1, 2, 3; \\ &\phi_{\text{ext}}(t) = \delta_{i1}\phi(t) + \phi_{\text{c}}, \end{aligned} \tag{1}$$

where $\phi(t)$ is a time-periodic magnetic driving acting on the first ring only and ϕ_{c} stands for the constant magnetic field which drives all rings. The coupling between rings is encoded in the matrix $\mathcal{M}_{ik}$ where off-diagonal elements are simply mutual inductances, between the ith and kth rings, whereas $\mathcal{M}_{ii} = \mathcal{L}$ is the self-inductance of the ring. For the co-planar alignment considered in this paper the mutual inductance is negative, i.e. it produces in the neighboring ring a current with the opposite direction to the one that is actually running in a given ring. The explicit form of $\mathcal{M}_{ik}$, depending only on the geometry (size, thickness and relative distance between the rings) of the system, is known [6, 7] and is of the form

$$\mathcal{M}_{ik} = f(r_{ik}), \tag{2}$$

where r_{ik} is the distance between centers of the ith and kth rings and [7]

$$f(r) = -\pi^2 R \left[\left(\frac{R}{r}\right)^3 + \frac{9}{4}\left(\frac{R}{r}\right)^5 + \frac{375}{64}\left(\frac{R}{r}\right)^7 \dots \right]. \tag{3}$$

We assume that each ring has a radius R. The stationary properties of this model, extended to the thermodynamic limit of an infinite array, have been studied in the series of papers [8]–[10].

The current I_k consists of two contributions: the so called persistent current I_k^{coh} and the dissipative current I_k^{nor},

$$I_k = I_k^{\mathrm{coh}} + I_k^{\mathrm{nor}} = I^{\mathrm{coh}}(\phi_k) + I^{\mathrm{nor}}(\phi_k). \tag{4}$$

What makes the non-superconducting mesorings essentially different is a highly non-trivial contribution of the persistent current related to the phase coherent electrons [4]. Here we assume that the ring is formed as a set of quasi-one-dimensional current channels. We can also assume that statistically one half of the channels accommodate an even number of electrons whereas the other half accommodate an odd number. Then the resulting coherent current is expressed by the relation [4]

$$I_k^{\mathrm{coh}} = I^{\mathrm{coh}}(\phi_k) = \frac{I_0}{2}\sum_{n=1}^{\infty} A_n(T)\left[\sin\left(2\pi n\frac{\phi}{\phi_0}\right) + \sin\left(2\pi n\left(\frac{\phi}{\phi_0}+\frac{1}{2}\right)\right)\right], \tag{5}$$

where the flux quantum $\phi_0 = h/e$ and I_0 is the maximal amplitude of the coherent current at zero temperature (which can be related to the size of the ring and the number of coherent electrons as discussed in detail in [4]). The temperature enters the current via the expansion coefficients:

$$A_n(T) = \frac{4T}{\pi T^*}\frac{\exp(-nT/T^*)}{1-\exp(-2nT/T^*)}\cos(k_{\mathrm{F}} l_x), \tag{6}$$

where k_{F} is the Fermi momentum, the characteristic temperature T^* is proportional to the energy gap Δ_{F} at the Fermi surface and l_x is the circumference of the ring.

At non-zero temperature, some of the coherent electrons become 'normal' electrons and they contribute to dissipative or Ohmic current. According to Ohm's law and the Lenz rule, this current reads

$$I^{\mathrm{nor}}(\phi_k) = -\frac{1}{R}\frac{\mathrm{d}\phi_k}{\mathrm{d}t} + \sqrt{2D_0}\,\Gamma_k(t). \tag{7}$$

In this expression, we take into account thermal, Johnson–Nyquist fluctuations of the Ohmic current. Current fluctuations are modeled as the Nyquist–Gaussian white noise

$$\langle \Gamma_k \rangle = 0, \qquad \langle \Gamma_k(t)\Gamma_i(s)\rangle = \delta_{ik}\delta(t-s), \tag{8}$$

where D_0 is the noise intensity.

From equations (1), (5) and (7) we obtain the evolution equation that governs the coupled dynamics of the magnetic fluxes. It is of the form [10]

$$\frac{1}{R}\sum_{k=1}^{3}\mathcal{M}_{ik}\frac{\mathrm{d}\phi_k}{\mathrm{d}t} = -(\phi_i + \phi_{\mathrm{ext}}(t)) + \sum_{k=1}^{3}\mathcal{M}_{ik} I^{\mathrm{coh}}(\phi_k) + \sqrt{2D_0}\sum_{k=1}^{3}\mathcal{M}_{ik}\Gamma_k(t). \tag{9}$$

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The above set of evolution equations has, however, a non-typical form: in terms of classical mechanics, if ϕ_k represents the particle coordinate, the left-hand side corresponds to unusual coupling of the 'flux velocity' $(\mathrm{d}\phi_k/\mathrm{d}t)$ degree of freedom (the interaction via velocities). The system (9) can be reformulated to a standard form by multiplying both sides by the elements $(\mathcal{M}^{-1})_{ni}$ of the inverse matrix of the inductances matrix. After summing over all i elements, one ends up with the equation

$$\frac{1}{R}\frac{\mathrm{d}\phi_n}{\mathrm{d}t} = -\sum_{i=1}^{3}(\mathcal{M}^{-1})_{ni}(\phi_i + \phi_{\mathrm{ext}}(t)) + I^{\mathrm{coh}}(\phi_n) + \sqrt{2D_0}\Gamma_n(t). \tag{10}$$

This form looks like a set of Langevin equations of motion for a system of overdamped particles in a potential related to the 'force' $I^{\mathrm{coh}}(\phi_n)$ acting on the kth particle and interacting linearly with each other. For a consistent description, we have to determine the noise intensity D_0. Using the above particle analogy and a Langevin type equation, we can infer that the intensity of thermal fluctuations is given by the relation [11]

$$D_0 = \frac{k_{\mathrm{B}}T}{R}. \tag{11}$$

The parameter k_{B} denotes the Boltzmann constant and T is the temperature of the system. This relation is in accordance with the classical fluctuation-dissipation theorem [11]. In other words, when the time-dependent driving is zero, $\phi(t) = 0$ in equation (1), the stationary state of the system has to be a thermodynamic equilibrium state or the Gibbs state. To prove this, one can write a Fokker–Planck equation corresponding to the Langevin equation (10). It fulfils the detailed balance principle [12] and therefore can be solved in a steady state which is indeed a Gibbs state. The readers can compare detailed studies presented in [9].

It is found to be very convenient to use the dimensionless version of any physical quantities. This provides the relations between energies and characteristic time scales for the system. The dimensionless form reads

$$\frac{\mathrm{d}x_n}{\mathrm{d}s} = -V'(x_n, s, T) - \sum_{i=1(\neq n)}^{3} \lambda_{ni}x_i + \sqrt{2D}\xi_n(s), \tag{12}$$

where the dimensionless flux is $x_n = \phi_n/\phi_0$ and the dimensionless time is $s = t/\tau$, where $\tau = \mathcal{L}/R$ stands for the relaxation time. The prime denotes the derivative with the respect to the first argument of the generalized potential $V(x_n, s, T)$, i.e. to x_n. The effective potential is given by

$$\begin{aligned} V(x_n, T) = {} & \tfrac{1}{2}a_n x_n^2 - b_n(s)x_n - i_0 \int \sum_{m=1}^{\infty} A_m(T)(p\sin(2m\pi x) \\ & + (1-p)\sin(2m\pi x + \tfrac{1}{2}))\,\mathrm{d}y. \end{aligned} \tag{13}$$

Other parameters read [4, 10]:

- coupling $\lambda_{ni} = \mathcal{L}(\mathcal{M}^{-1})_{ni}$,
- nth diagonal element $a_n = \mathcal{L}(\mathcal{M}^{-1})_{nn}$,
- externally induced fluxes $b_n(s) = \gamma_n\phi_{\mathrm{ext}}(s)/\phi_0$, $\gamma_n = \mathcal{L}\sum_{i=1}^{3}(\mathcal{M}^{-1})_{ni}$,

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- characteristic current $i_0 = \mathcal{L}I_0/\phi_0$,
- zero-mean, δ-correlated, rescaled noise $\xi(s) = \sqrt{\tau_0}\,\Gamma(\tau s)$,
- intensity of noise $D = k_\mathrm{B}T/2\varepsilon_0$, with $\varepsilon_0 = \phi_0^2/2\mathcal{L}$.

3. Numerical experiment

In order to establish the influence of the external time-periodic driving

$$b_n(s) = \delta_{1n} a \cos(\omega s) + b_c \tag{14}$$

on the signal propagation within the set of three mesoscopic rings in a co-planar one-dimensional array (9), we have carried out extensive numerical simulations. We have integrated the Langevin equation (12) by employing the stochastic Euler method with a time step depending on the noise strength and the period of the external driving. For the initial condition of the phase (coordinate) $x(t)$, we used a uniform distribution on the interval $x \in [-2, 2]$. All quantities are averaged over at least 10^5 periods of the external driving.

We have varied two parameters describing the external periodic forcing: the amplitude a and the frequency ω. Other parameters used in simulations were set as follows: $D = 0.1$, $b_c = 0.1$, $p = 0.5$. The relaxation time $\tau = 1$ is one of the relevant time scales for the system (12). Another characteristic time is the period $\mathcal{T} = 2\pi/\omega$ of the external driving. The interplay between these two times, i.e. between relaxation and excitation of the system (12), can lead to phase shift in consecutive rings, amplification and rectification of the external signal.

3.1. Phase lag

Depending on the period (or angular frequency) of the periodic stimulus, the phase lag can be observed in consecutive rings. We can define the response times of the given ring as the delay time between:

τ_1: the response of the first ring and the external time-periodic stimulus,

τ_2: the response of the second ring and the signal from the first ring,

τ_3: the response of the third ring and the signal from the first ring.

One should keep in mind that the dynamics of the magnetic flux of any ring depends on the dynamics of other rings from the set via the mutual inductances. In figure 2 we present four cases for slow ((a) $\omega = 0.01, \mathcal{T} = 200\pi$), moderate ((b) $\omega = 0.05, \mathcal{T} = 40\pi$) and fast ((c) $\omega = 0.5, \mathcal{T} = 4\pi$, (d) $\omega = 1, \mathcal{T} = 2\pi$) drivings. The strength of the external force was set to $a = 5$ for all cases depicted in figure 2. If the external force (which acts on the first ring only) is slow (small ω or large $\mathcal{T}$), the change of magnetic flux within one period would proceed in the same manner in every single ring from the set. This situation is presented in panel (a) of figure 2. The three times that describe the delay have rather small values ($\tau_1 = 0.0012$, $\tau_2 = 0.0044$ and $\tau_3 = 0.0011$ in units of the external driving period). This is rather easy to understand: for a slow enough driving the system

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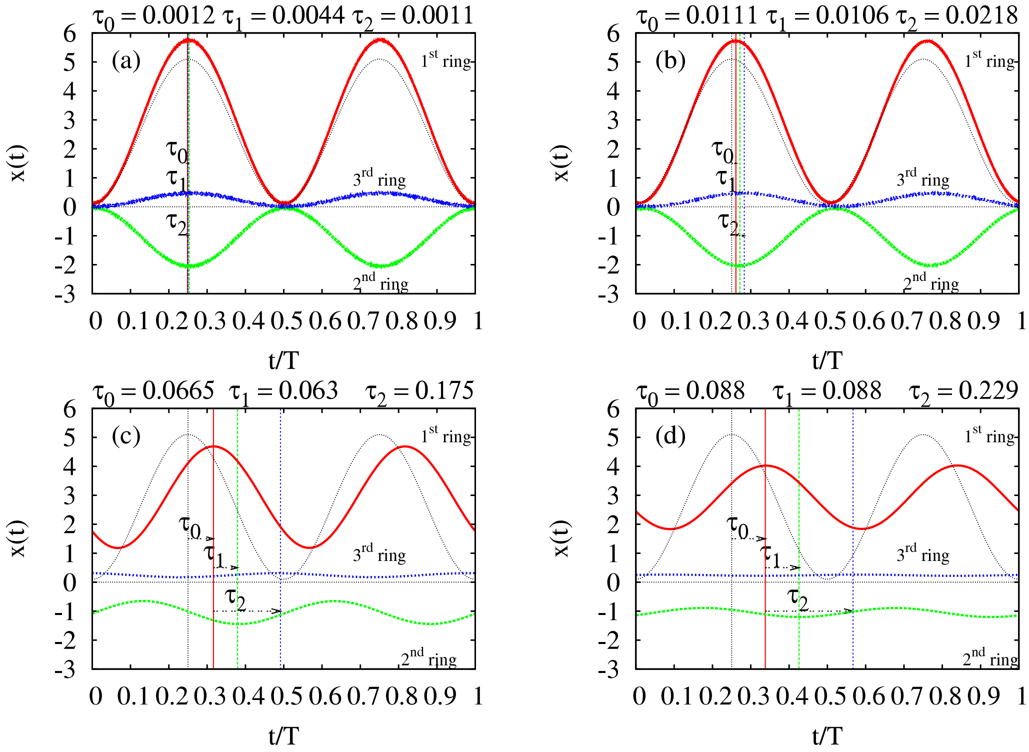

Figure 2. The mean magnetic flux is presented for one period $\mathcal{T}$ of the external alternating magnetic force for all three rings in a chain (first ring—red, second ring—green, third ring—blue, signal—dotted gray). Vertical colored lines denote the position of extrema of the flux for the corresponding ring. Please note that there is no phase lag for the slow driving situation for any of the three rings (panel (a): $\omega = 0.01$) and it increases with increasing frequency (panels (b): $\omega = 0.05$, (c): $\omega = 0.5$, (d): $\omega = 1$).

has enough time to recover from excitation and one can easily see that there is almost no delay in the dynamics of the flux for every ring.

If, however, we increase the frequency, rings will not be able to react fast enough and the phase lag between the magnetic fluxes from the consecutive rings will eventually appear. This case is shown in the next three panels of figure 2. Panel (b) corresponds to the moderate frequency of the external time-periodic force and it is obvious upon inspection that there is already a small time lag ($\tau_1 = 0.0111$, $\tau_2 = 0.0106$ and $\tau_3 = 0.0218$). If we increase the frequency even more, like for the situations shown in panels (c) and (d), the time lag between fluxes increases.

For higher frequencies of the driving force we are able to calculate only the time delay between the signal and the first ring τ_1 due to the rectification of the magnetic flux (see the response of the third ring in panel (d) and section 3.3 for details). For the slow driving frequencies the response time τ_0 equals zero (or almost zero). For fast signals it reaches the value $\tau_1 = 0.125\mathcal{T}$; see figure 3. This feature is independent of the driving strength a. Is this a general rule for the system (12)? To answer this question we need to study different set-ups.

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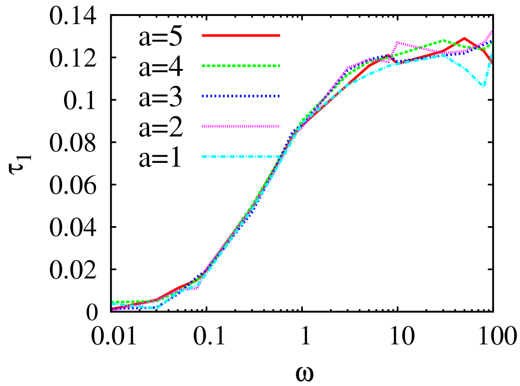

Figure 3. The phase lag between the external signal and the magnetic flux induced in the first ring τ_1 is presented versus driving angular frequency ω for different driving strengths $a = 1, 2, 3, 4, 5$. The delay reaches its maximum $\tau_1 \approx 0.125\mathcal{T}$ for fast alternating forces and is rather independent on the strength a.

3.2. Vibrational amplification

A slowly driven ring can exhibit another interesting phenomenon—vibrational enhancement of the magnetic flux. A similar feature was already reported for a similar system of periodically driven mesoscopic cylinders [13] but showing rather typical bell-shaped dependence on the noise strength, revealing well known stochastic resonance [14]. The amplification of the signal appears as a result of an interplay between coherent and Ohmic currents flowing in a ring as well as periodic driving and thermal noise; cf panels (a), (b) in figure 2. We can refer to this feature as a *vibrational amplification* of the magnetic signal. The ratio of the amplitude of the mean induced magnetic flux on the given ring and the amplitude of the external signal defines the spectral amplification factor

$$A_f = \frac{|\langle x(t)\rangle|_{\max}}{a}. \tag{15}$$

In figure 4 we plotted the amplification factor versus frequency of the signal for all rings and for five different amplitudes $a = 1, 2, 3, 4, 5$. One can note that the behavior of the magnetic flux weakly depends on the driving strength and for the first ring it has a maximum ($A_f \simeq 1.14$) for slow forces and tends to $A_f \simeq 0.57$ for fast drivings. In figure 4 we showed the amplification/reduction of the signal in the rings. Mutual inductance between nodes is quite weak and the suppression of the signal in the next two rings is rather intuitive. The possibility of producing an amplification factor of value at least close to the amplitude of the driving in the rings *not* driven directly by the stimulus remains an open question.

3.3. Flux rectifier

If the device relaxes from the excitation in a finite, non-zero time it is understood that for very fast repetitions of excitation it will not be able to recover to the minimum energy state, but will rather stay excited. If we apply fast driving to the system (12) we will recognize the constant magnetic flux on all three rings; see the situation in panel (a) in

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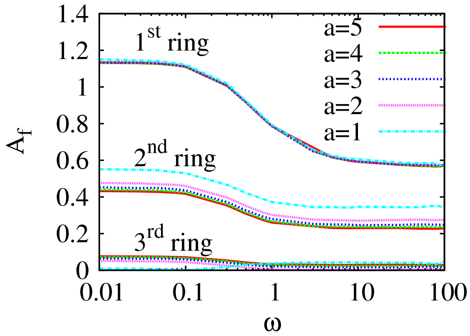

Figure 4. Vibrational activation of the signal in the first ring. For five different force strengths $a = 1, 2, 3, 4, 5$ we present the amplification factor (15) versus the driving frequency. Note its independence of the driving strength.

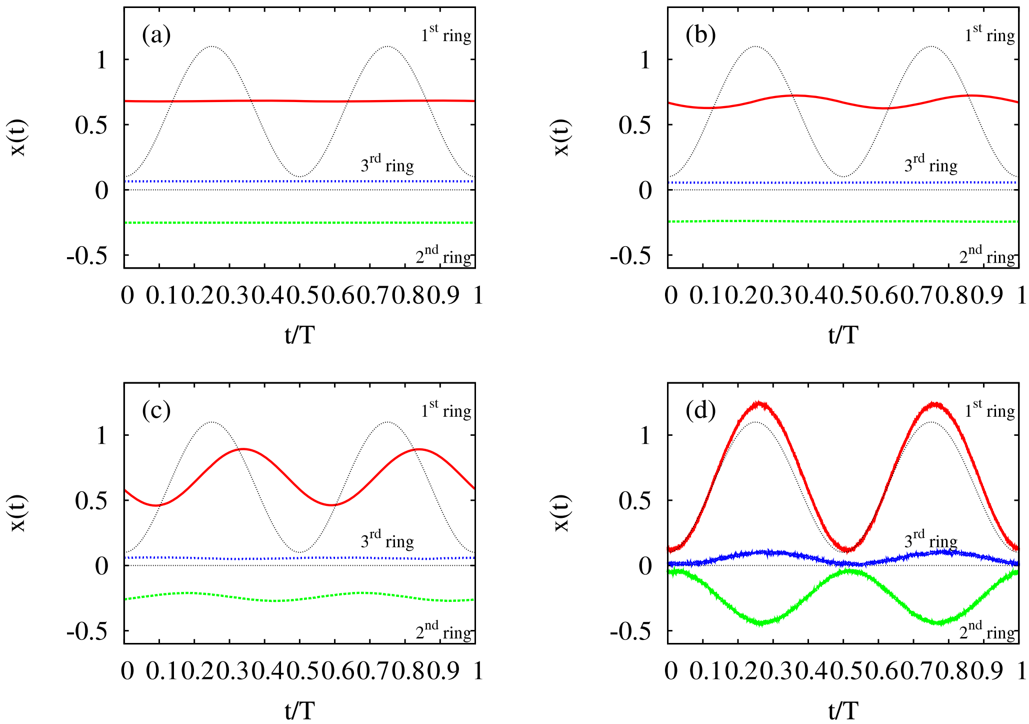

Figure 5. Rectification of the alternating signal by use of a chain of mesoscopic rings is presented for four different frequencies of the external magnetic field $\omega = 80, 5, 1, 0.05$. The force strength is set to $a = 1$. Note that for very fast drivings (panel (a)) all rings produce constant flux. If we decrease the frequency we will eventually destroy this feature in consecutive rings for slower frequencies; see panels (b)–(d).

figure 5 for $\mathcal{T} = \pi/40$. For slower fields this effect should disappear. In our situation this is actually happening only for the dynamics of the first ring. The second and third rings still produce constant (to some extent) magnetic flux for slower fields also. For $\mathcal{T} = 2\pi/5$ (panel (b)) the last two rings still produce constant magnetic field, while the first ring follows in an alternating manner. By decreasing the frequency even more ($\mathcal{T} = 2\pi$) we induce the periodic response in the first and second rings but are still able to keep the signal from the third constant (see panel (c)). Of course, by applying a force of even slower modulation we are not able to rectify the flux in any ring (panel (d)). But is it entirely impossible? Probably there is a possibility of rectifying even very slow magnetic fields just by applying it to the chain of more mesoscopic rings, not just these few. This is however not straightforward. The mutual inductance, which is the only connector between the nodes, is far too weak to carry the information over more than a few rings. It is then necessary to study rings with additional diamagnetic cores or, on the other hand, to try to put the vibrational enhancement to work to amplify the signal. Either way, we still are not able to answer the question of whether we are able to rectify moderate time-periodic magnetic fields by using a 1D chain of mesorings.

4. Conclusions

The problem of signal transmission in a co-planar network of mesorings addressed in this work exhibits various non-trivial properties. Here we focused on the most spectacular examples: the phase lag, vibrational enhancement of magnetic flux and the possibility of a rectification of magnetic signals. As the system under consideration is inductively (i.e. rather weakly) coupled, the effective application of the ideas presented requires careful preparation. We have shown that under suitable choices of the periodic driving of the first ring in the array, a phase lag between input and the output signal is achieved. It is also shown that the signal can be both significantly amplified and effectively rectified. Such effects are not only interesting from the point of view of basic research but also due to their potential applicability in building ‘flux guides’ or ‘magnetic fibers’ transmitting signals at the nanoscale.

There are still many open problems existing and waiting for a solution. Some of them are very general (e.g. the influence of quantum thermal fluctuations which are important at very low temperatures [15] and correct fluctuation–dissipation relations in this regime), some are system oriented (e.g. whether the number of rings is relevant for the flux enhancement), but all of them are challenging for the future study of mesoscopic systems of cylindrical symmetry both from theoretical and from experimental points of view.

Acknowledgments

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References

[1] Ibbotson L, 1988 *The Fundamentals of Signal Transmission, Optical Fibre, Waveguides and Free Space* (Oxford: Butterworth–Heinemann)
[2] Mohanty P, 1999 *Ann. Phys. Lpz.* **8** 549
[3] Zipper E, Kurpas M, Szelag M, Dajka J and Szopa M, 2006 *Phys. Rev.* B **74** 125426
[4] Dajka J, Łuczka J, Szopa M and Zipper E, 2003 *Phys. Rev.* B **67** 073305
[5] Dajka J, Machura Ł, Rogozinski S and Łuczka J, 2007 *Phys. Rev.* B **76** 045337
[6] Jackson J D, 1975 *Classical Electrodynamics* 2nd edn (New York: Wiley)
[7] Wang J and Ma Z S, 1995 *Phys. Rev.* B **52** 14829

[8] Dajka J, Łuczka J, Szopa M and Hänggi P, 2005 *Phys. Status Solidi* b **242** 196
[9] Łuczka J, Dajka J and Hänggi P, 2005 *Acta Phys. Polon.* B **36** 1693
[10] Łuczka J, Dajka J, Mierzejewski M and Hänggi P, 2005 *AIP Conf. Proc.* **800** 197
[11] Risken H, 1984 *The Fokker–Planck Equation* (Berlin: Springer)
[12] Gardiner C W, 1983 *Handbook of Stochastic Methods* (Berlin: Springer)
[13] Łuczka J, Dajka J, Mierzejewski M and Hänggi P, 2006 *Acta Phys. Polon.* B **37** 1605
[14] Gammaitoni L, Hänggi P, Jung P and Marchesoni F, 1998 *Rev. Mod. Phys.* **70** 223
[15] Machura L, Kostur M, Hanggi P, Talkner P and Łuczka J, 2004 *Phys. Rev.* E **70** 031107

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